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(Stationary Model of Water Movement in a
Stratified Strait (using Bosporus [Istanbul Boğazi]
and Other Straits as Examples))

by

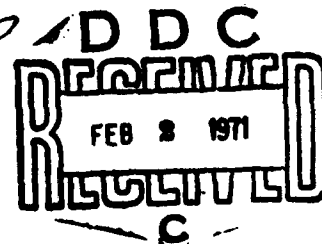D. M. TOLMAZIN

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ABSTRACT

Two-layer water stratification in straits, such as the Bosporus,
Bab-el-Mandeb, and Gibraltar, is discussed. Conclusions, based
on observed data, are compared with the results of mathematical
calculations by considering the position of the interface between
the surface and bottom currents, their velocities, slopes and
other phenomena. The two opposite currents appear to be active
constantly, although the velocity of the bottom current sometimes
is extremely small or almost nonexistent. When this occurs, the
water-level decrease in the Bosporus, between the Marmara and
Black Seas, should exceed 36cm. According to observations and
calculations, such cases are rare exceptions and are limited to
brief periods.

Translator

STATIONARY MODEL OF WATER MOVEMENT IN A
STRATIFIED STRAIT (USING BOSPORUS [ISTANBUL BOGAZI]
AND OTHER STRAITS AS EXAMPLES)

The hydrological regime of southern seas, such as the Black,
Mediterranean and Red Seas, is largely determined by water exchange
through connective straits. The main properties of these straits
are a two-layer structure of currents and a considerable stratifica-
tion of water, which are determined by differences in the components
of fresh water portions of southern seas from surrounding portions
of the World Ocean.* Accordingly, it is useful to consider the
problem of water movement as part of the system made up of a sea and
a strait and affected by certain external factors, such as wind,
heat and water balance. However, the solution to such a problem
is associated with nearly unsurmountable mathematical difficulties.

This paper discusses a more simplified problem -- namely, the
investigation of currents and densities in the Bosporus and other
straits, on the condition that differences in densities and water
levels at end points and at a given wind over the strait are known.
Within such a framework the solution of a number of important
practical and scientific problems is possible, notably to evaluate
conditions at which the lower level water masses become stratified
in straits. The warm and saline water flowing along the bottom
from mediterranean seas exercises considerable influence on the
biology of the deep water of oceans and inland seas. This process
is of great significance in the formation of salinity and biologi-
cal water structure of the Black Sea.

The hydrodynamic model of water movement in a strait has a number
of characteristics. Investigations of steady state currents in
straits, which are based on two-layer (Defant, 1961; Tolmazin,
1962) and multi-layer density models (Takano, 1958), are associated
with unavoidable deficiencies. Thus, the existence of a certain
density stratification and absence of density transport by currents
must be assumed. As a result, the processes associated with a   /19
diffusive vertical water transport were not investigated. According
to K. Takano, the omission of this diffusive vertical water transport
has lead to certain discrepancies: the theoretical intensity of
integral transport of mediterranean water at Gibraltar differs from
the observed value.

In order to eliminate these discrepancies and form a more accurate
concept on mechanics of water movement in stratified straits, we
discuss the stationary model of currents and density field by means
of a nonlinear equation of turbulent diffusion (Tolmozin, 1964).

> *This property enables us to consider such straits
> as being stratified. Zubov (1956) calls them straits
> with density-induced water exchange.

The main simplifications of our problem (omission of nonlinear
members in motion equations and the neglect of vertical advection
in diffusion equation) are based on numerical evaluations of the
relative significance of these processes in the formation of major
fields. These calculations made, for the Bosporus, Dardanelles
(Canakkale Bogazi) and other straits by analyzing the Pritchard
Calculations (1954, 1956) for stratified estuaries agree with the
latter.

You should note that the suggested model is reminiscent of the
Rattry and Hansen (1962) model, plotted for a stratified estuary.
These writers, however, did not account for one of the important
moving forces in a strait — namely, the longitudinal water level
drop.

<div align="center">

Definition of the Problem on Water
Movement in a Stratified Strait

</div>

Dynamics of processes occurring in sea straits is determined (1) by
the action of known climatic factors on the sea surface, which
create the drop of sea levels and densities at the ends of straits,
and (2) by the action of wind tension on the water expanse. To
find the stationary fields that are formed under these conditions,
we use a simplified system of equations of motion and turbulent
diffusion:

$$\mu \frac{\partial^2 u}{\partial z^2} = \frac{\partial p}{\partial x}; \tag{1}$$

$$\frac{\partial p}{\partial z} = g\rho; \tag{2}$$

$$u \frac{\partial \rho}{\partial x} = k \frac{\partial^2 \rho}{\partial z^2}, \tag{3}$$

where u = velocity projection on x axis; ρ = density; p = pressure; /20
μ and k = coefficients of turbulent viscosity and diffusion,
assumed to be constant; g = acceleration of gravity. The x axis
of a rectangular system of coordinates is directed along undisturbed
surface to the seaward where the density is smaller; the z axis runs
vertically downward. The given strait is assumed to have a
rectangular intersection, a constant width and variable depth, h.

We introduce deviation of density ρ' from a certain constant value p_*:

$$\rho' = \rho - \rho_*. \qquad (4)$$

Eliminating pressure, p, by cross differentiation, we find in lieu of (1)–(3) that:

$$\mu \frac{\partial^3 u}{\partial z^3} = g \frac{\partial \rho'}{\partial x}; \qquad (5)$$

$$u \frac{\partial \rho'}{\partial x} = k \frac{\partial^2 \rho'}{\partial z^2}. \qquad (6)$$

Equation (5) enables us to introduce an auxiliary function Q determined by equations:

$$u = \frac{1}{\mu} Q_x; \qquad (7)$$

$$\rho' = \frac{1}{g} Q_{zzz}. \qquad (8)$$

Here the subscripts denote differentiation on the basis of the corresponding variable.

Introducing (7) and (8) into (6), we arrive at the basic equation

$$Q_x Q_{zzzz} = \mu k Q_{zzzzz}. \qquad (9)$$

Boundary conditions for the function, Q, can be derived from conditions of velocity and density:

$$\mu \left(\frac{\partial u}{\partial z} \right)_{z=-\zeta} = Q_{zz}(x_1, -\zeta) = -T; \qquad (10)$$

$$u|_{z=h} = Q_z(x, h) = 0; \qquad (11)$$

$$\left(\frac{\partial \rho'}{\partial z} \right)_{z=-\zeta} = Q_{zzzz}(x, -\zeta) = 0; \qquad (12)$$

$$\rho'|_{z=-\zeta} = \frac{1}{g} Q_{zzz}(x, -\zeta). \qquad (13)$$

where ζ = excess water level over the disturbed surface;
     T = wind friction against water.

Condition (13) denotes that the density distribution at surface
is given. Consider that this change occurs in accordance with
the exponential law:

$$\rho'(x, -\zeta) = \rho_n' \left(\frac{x}{l}\right)^e,$$

where l = length of strait; ρ_n' = const. at surface. The lacking
fifth condition with regard to z can be determined from the continuity
equation at a given rule of water level change, $\zeta(x)$

$$\frac{d}{dx}\int_{-\zeta}^{h} u\,dz = \frac{d}{dx}\int_{-\zeta}^{h} Q_x\,dz.$$

With a great degree of accuracy, we can write

$$\frac{d}{dx}\int_{0}^{h} Q_x\,dz + \frac{d}{dx}[\zeta Q_x(x, 0)] = 0. \qquad (14)$$

Equation (9) requires one condition with regard to x. A given
density at one end of the strait (x = x_1) in the form of the known
function

$$\rho' = \rho'(x_1, z).$$

can be used as such a condition.

We examine the dimensionless variable by assuming that

$$\left.\begin{array}{c} x = l\xi, \quad z = H_0\bar{z}, \quad Q = Q_0\bar{Q}, \quad h = H_0\bar{h} \\ \rho' = \rho_0'\bar{\rho}', \quad u = u_0\bar{u}, \quad \zeta = \zeta_0\bar{\zeta} \end{array}\right\} \qquad (15)$$

where Q_0, H_0, u_0, ρ_0' and ζ_0 = characteristic values of the
corresponding magnitudes. Accounting for (15), we arrive at the
basic equation in a dimensionless form:

$$\varepsilon \bar{Q}_{\xi\xi\xi\xi} = \bar{Q}_\xi \bar{Q}_{z\xi z}, \qquad (16)$$

where $\varepsilon = \dfrac{\rho k l^2}{Q_0 H_0^2}$ = a small dimensionless parameter.

Due to the smallness of ζ, the surface conditions can be transferred
to z = 0:

$$\left.\begin{array}{c} \bar{Q}_{\bar{z}}(\xi, 0) = \dfrac{\tau}{\lambda}, \quad \bar{Q}_\xi(\xi, \bar{h}) = 0 \\[2mm] \bar{Q}_{zzz}(\xi, 0) = 0, \quad \bar{Q}_{zz\xi}(\xi, 0) = \gamma\xi^n \\[2mm] \dfrac{d}{d\xi}\int_{0}^{h} \bar{Q}_\xi\,d\bar{z} + \dfrac{\zeta_0}{H_0} \cdot \dfrac{u_0}{\Delta U_0} \cdot \dfrac{d}{d\xi}[\bar{\zeta}\bar{Q}_\xi(\xi, 0)] = 0 \end{array}\right\}, \qquad (17)$$

where $\tau = -\dfrac{TH_v}{u_0 \mu}$, $\gamma = \dfrac{g\rho'_\mu H_0^3}{Q_0}$, $\lambda = \dfrac{g\rho' H_c^2}{lT}$, ΔU_v = a typical
change of the mean vertical speed.

The typical scale of Q can be expressed by known magnitudes,
depending on the predominant current component, which may be
either a wind or density component. In case the wind-induced
currents, as defined in (17) predominate,

$$Q_0 = l H_0 T. \qquad (18)$$

can be used.

If the currents are determined mainly by a longitudinal density
drop, we have

$$Q_0 = g\rho'_v H_v^3. \qquad (19)$$

Because the main current system of straits is created by nonmechan-
ical causes, expression (19) should be used in the evaluation of
Q_0. Thus,

$$s = \dfrac{\mu k l^2}{g\rho'_0 H_0^5}.$$

Assume that $H_0 = 7.2 \cdot 10^3$ cm; $l - 3 \cdot 10^6$ cm; $\rho'_0 = 10^{-2} \, v \cdot cm^{-3}$; $\mu = 300 \, v$
$\cdot cm^{-1} \cdot cek^{-1}$; $K = 5.3$ cm $\cdot cek^{-2}$ (conditions in the Bosporus). Under
these conditions $\varepsilon = 0.763 \cdot 10^{-4}$. For other straits, where H_0 is
larger, ε is still smaller (Table 1).

The finite magnitudes of the problem (typical depth, velocity,
etc.) have been closer with the design that the size of the area
and the magnitude of the nonuniform member were near unity. The
dominant behavior of $\bar{Q}$ in the equation appeared to be a small
parameter at the leading derivative. This fact enables us to
formulate our problem as that of a boundary layer. On the basis
of such an approach, the character of the vertical profile, $\bar{Q}$, can
be studied without solving the equation (16).

The concept "boundary layer," which usually has been associated   /23
with a phenomenon occurring near a body when the magnitudes of
Reynold's numbers are large, is now associated with many aspects
of applied mechanics. Such a generalization is discussed by G.
F. Carrier (Karrier) (1955). The boundary layer denotes a zone
where the given function increases at a rapid rate; therefore, at
a certain distance from the boundary the value of the function

TABLE 1

Parameter	Bos-porus	Bab-el-Mandeb	Gibraltar
h_1, м	48	150	350
h_2, м	72	350	—
H_0, м	72	400	500
l, км	30	80*	100**
ξ_1	0,861	0,225	1,094
ξ_2	1,861	1,225	2,094
a	—0,525	—0,500	—0,550
m	0,722	0,791	0,666
μ, г/см·sec	300	200	150
k, см/sec.	5,3	64	100
$\epsilon\cdot10^5$	0,762	0,707	0,050
ρ_0, г/см³	1,0080	1,0225	0,0265
u_0, см/sec	100	100	100
ΔU, см/sec	5,0	0,3	0,5
r_0, см	10	3	1
$\lambda\cdot10^2$	0,41	3,33	8,33

NOTE:

*Distance from Cape Raheita to the deep portion of the Gulf of Aden.

**Distance from the east end of strait to the deep portion of the Atlantic Ocean.

becomes "smooth" (inner area). The /23 main property of the boundary layer, according to Carrier, is the small parameter at the leading derivative in the main equation.

In our case, the existence of boundary layers is possible near the surface and bottom. Let us appraise the thickness of these layers (areas of sharp increase of function $\bar{Q}$). For this purpose, we change the scale of coordinates that are normal to the boundaries. Because ζ is very small, we assume that

$$\chi_1 = \varepsilon^n \bar{z}, \qquad (20)$$

for bottom ($\bar{h} = \varphi(\xi)$ = known function

$$\chi_2 = \varepsilon^n [\bar{z} - \varphi(\xi)].$$

The n value in the coordinates of boundary layers is selected so that the order with respect to ε in the left part of (16) coincides with the order with respect to ε in the corresponding part of $Q_E Q_{Ezzz}$. Introducing these coordinates into (16), we can assume that n = -1/3. Thus, the thickness of the boundary layer has the order of $\varepsilon 1/3 H_0$.

By introducing the typical values ε and H_0, calculated for various straits, the thickness of boundary layer appears to be nearly equal to 10m.

There is no need to discuss separately the boundary layer and the inner area because a special solution to (16) is necessary. We note that the neglect of a member with parameter leads to degeneration of equation (16). This is evidently associated with the character of motion in stratified straits: in a two-layer current, the velocity u (or Q_E) becomes 0. Evidently, in the center of the inner area we have to deal with small left and right-hand parts of equation (16).

The qualitative analysis enables us to describe the behavior of function $\bar{Q}$ in vertical direction: a sharp increase of function Q at the bottom to a certain maximum is replaced by a zone where the value of the function decreases uniformly and the $\bar{Q}$ reaches 0, then increases again, with another sign, at the surface. Thus, the

small parameter ε determines the appearance of boundary layers
when defining the process with the aid of supplementary function   /24
Q. The thickness of boundary layers (their wash-out) depends,
by means of ε, upon the value of viscosity and diffusion coefficients.

Automatic Model Solution to the Main Equation

The suggested method for solving (16) and (17) has been successfully
used in a number of projects dealing with ocean circulation. The
main feature of it is the transformation of coordinates at which
the equation can be reduced to a simple form. The solution of this
equation is a function whose profile has the property of similarity
for all intersections of the area, differing only in the scale.

$\bar{Q}$ can be expressed as follows:

$$\bar{Q} = \varepsilon M(\xi) f(\eta), \qquad (21)$$

where

$$\eta = \bar{z} \xi^a m^{-1}.$$

Here α and m are unknown parameters determined by the geometric
shape of the given strait. To solve $0 \leq h \leq 1$, it is necessary to
define the pattern of change in the depth of the given strait:

$$\bar{h} = m\xi^{-a}. \qquad (22)$$

It is evident that the actual values of $\bar{h}$ and η will be materialized
when ξ varies within $\xi_1 \leq \xi \leq \xi_2$, where ξ_1 and ξ_2 are dimensionless
coordinates of the beginning and the end of the strait, whereby
$\xi_1 > 0$. The variation of parameters α, m and ξ, enables us to
determine quite accurately the general slope of bottom in a number
of straits.

The small parameter of factor ε in (20) does not mean that $\bar{Q}$ is
a small value. Here the magnitude scale has been artificially
changed (order f equals $\underline{1}$) in order to eliminate ε in the equation
defining f. ε

One can readily see that one of the variables can be excluded
from (16) only when

$$M = \xi^{2a+2}. \qquad (23)$$

Accounting for (23) and introducing (20) into (16), we have the following simple equation for f:

$$f^{V} = m^{2}x^{2}\tau_{1}^{2}f^{IV}f^{I} + a\tau_{1}f^{IV}f + b\tau_{1}f^{III}f^{I} + cf^{III}f. \qquad (24)$$

The Roman numbers denote in this case the order of differentiation with respect to η.

Boundary conditions for f become   /25

$$\frac{f^{I}(0)}{\varepsilon} = \frac{m\tau}{\varepsilon^{2}(3\alpha+2)\,\xi^{3\alpha+1}} = D_{1} = \text{const};\qquad (25)$$

$$f^{I}(1) + sf(1) = 0; \qquad (26)$$

$$f^{IV}(0) = 0; \qquad (27)$$

$$\frac{f^{III}(0)}{\varepsilon} = \frac{m^{3}\rho_{n}^{'}\xi^{\theta}}{\varepsilon^{2}\rho_{0}^{'}\xi^{5\alpha+2}} = D_{2} = \text{const}; \qquad (28)$$

$$\int_{0}^{1} [sf + \eta f^{I}]\,d\eta + s\varkappa f(0) = 0. \qquad (29)$$

In (24)—(29) the following designations are introduced:

$$a = m^{2}\alpha\,(2\alpha+2); \quad b = m^{2}\alpha\,(5\alpha+2); \quad c = m^{2}\,(2\alpha+2)\,(5\alpha+2);$$

$$s = \frac{2\alpha+2}{\alpha}; \quad \varkappa = \frac{u_{0}n\zeta_{0}}{\Delta U_{0}mH_{0}}.$$

In order to preserve the given character of the process, it is necessary to impose certain restrictions on the longitudinal distribution of density at surface, and tangential wind pressure and surface:

$$\tau = \tau_{0}\xi^{3\alpha+2}, \quad \rho^{'}(\xi_{1}, 0) = \rho_{n}^{'}\xi^{5\alpha+2}, \quad \zeta = n\xi^{-\alpha}, \qquad (30)$$

Here n = proportionality coefficient determined by means of ξ values at the end of a given strait.

It follows from (29) that $\varkappa = 0$ is applicable when the loss of water in the given strait equals zero (losses of surface and bottom currents are equal).

Let us plot solution for (24)-(29), by expanding f(η) in series of ε degrees. Assume

$$f = \varepsilon \sum_{n=0}^{\infty} \varepsilon^{n} f_{n}. \qquad (31)$$

By introducing (31) into (24) and equalizing the coefficients
at equal powers of ε, we arrive at the following system of
equations: $f_0^V = 0$;

$$f_1^V = m^2 \varkappa^2 \tau_i^2 f_0^1 f_0^{IV} + a\eta f_0^{IV} f_0 + b\eta f_0^1 f^{III} + c f_0 f^{III};$$

$$f_2^V = m^2 \varkappa^2 \tau_i^2 (f_0^1 f_1^{IV} + f_1^1 f_0^{IV}) + a\eta (f_0 f_1^{IV} + f_1 f_0^{IV}) +$$

$$+ b\eta (f_0^1 f_1^{III} + f_1^1 f_0^{III}) + c (f_0 f_1^{III} + f_1 f_0^{III})$$

Accounting for the boundary conditions, we find a solution for the   /26
zero and first approximations:

$$f_0 = A_0 + \sum_{i=1}^{4} A_i \frac{\eta^i}{i!};$$ (32)

$$f_1 = B_0 + A_2 \sum_{i=1}^{8} B_i \frac{\eta^i}{i!},$$ (33)

where

$$A_0 = \frac{1}{24s(3\varkappa+2)} [(s+3) D_2 - 12 (s+1) D_1];$$

$$A^1 = D_1;$$

$$A_2 = -\frac{1}{4(s+2)(3\varkappa+2)} [(s+3)(4\varkappa+3) D_2 + (s+1)(2\varkappa+1) D_1];$$

$$A_3 = D_2;$$

$$B_0 = \frac{A_3}{s(3\varkappa+2)} \sum_{i=5}^{8} A_{i-5} \frac{[(i-5) b + c] (s+i)(i-2)}{(i+1)!};$$

$$B_2 = -\frac{6A_0}{(s+2)(3\varkappa+2)} \sum_{i=5}^{8} A_{i-5} \frac{[(i-5) b + c][i(\varkappa+1) + \varkappa](s+i)}{(i+1)!};$$

$$B_{i+5} = (ib+c) A_i. \quad A_1 = B_1 = B_3 = B_4 = 0.$$

The subsequent approximations in which high powers of ε occurs,
do not introduce substantial variations to the distribution of
density and velocity.

The Effect of External Factors on Vertical Distribution
of Density and Velocity in a Strait

To calculate the vertical velocity profile and density distribution
with the first approximation $(f_0 + \varepsilon f_1)$, we assume that $\alpha = -0.525$;
$m = 0.722$; $\varepsilon 0.763 \cdot 10^{-4}$.

Figure 1 shows the vertical velocity profiles in windless periods.
The velocity values of surface and bottom currents vary proportion-
ally to the density gradient in longitudinal direction.

The effect of water level decrease on current velocity is shown
in Fig. 2 (the other factors are assumed to be constant). With an
increase in longitudinal slope between seas directed against density
gradient, the velocity of the upper current and the depth of inter-
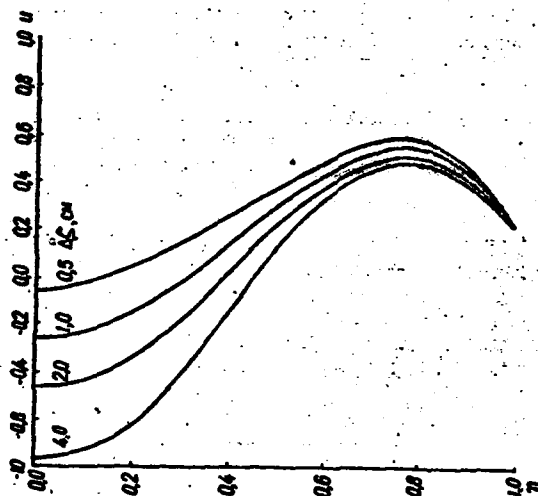face increase, while the velocity of the lower current decreases.

/27

FIG. 2. The Effect of Water Level
Decrease on Vertical Profile of
Current Velocity in Perfectly Calm
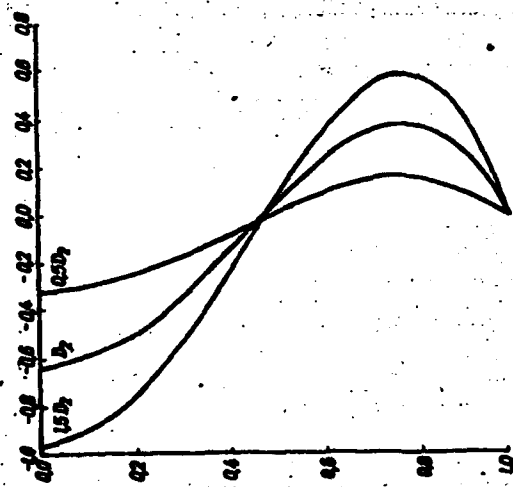Weather and at Constant D_2.

FIG. 1. The Effect of Density
Decrease, D_2, on the Vertical
Profile of Current in Perfectly
Calm Weather ($D_2=4.31 \cdot 10^7$; $X=0.1$;
$\omega=0.0$ m/sec).

Winds exercise a somewhat different influence on the vertical
velocity profile (Fig. 3). With a wind blowing against the
direction of density decrease, the velocity of the upper current
increases. Simultaneously, the velocity of the lower current
increases (as with an increase in the density gradient). This
characteristic is associated with the assumed stationary nature
of the process: preservation of the constancy of volume in seas
at various D_2 and W and fixed $\Delta\zeta$ must be associated with a definite
relationship between the losses in both the upper and lower currents.
As seen from Fig. 3, the velocity of wind-induced currents is
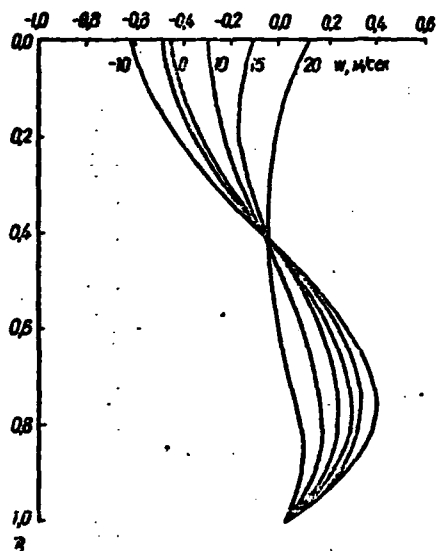limited.

The greatest variations in
current regime are usually
associated with wind-induced
de-levelling of free surface
in pre-tidal sea areas. This
phenomenon, based on observations,
is supported by formulae derived
in this paper. Indeed, the wind
action in straits is smaller than
gradient forces, but the density
difference changes little, mainly
due to seasonal variations in the
component of thermal and salinity
balance of the seas. Therefore,
the velocity values and thickness
of water layers, involved by the
upper and bottom currents, depend
mainly on the slope of sea level
in the given strait.

FIG. 3. The Effect of Longitu-
dinal Wind Component on Velocity
Profile at a Constant Sea Level
Decrease ($\Delta\zeta$ = 3cm) and Density
Gradient (sign minus denotes
that the wind blows in the
direction of surface current).

The formation of a three-layer
structure of currents is possible
with winds blowing against the
slope of free surface (see Fig. 3,
ω = 20 m/sec). Such a phenomenon
has been observed several times
in Bab-el-Mandeb Strait when the
currents near the surface or
bottom have the same sign and

countercurrent flows in the intermediate layer (Muromtsev, 1960;
Thompson, 1939).

Two approximations lead to the following formula of density deviation:

$$\bar{\rho}' = \varepsilon^2 m^{-2\xi^{5\varkappa+2}} D_2 \left\{ 1 + \frac{m^2 \varepsilon D_2 (5\varkappa+2)}{24 (3\varkappa+2)} \times \right.$$

$$\times \left[\frac{1}{2} \eta^3 - \frac{4\varkappa+3}{4} \eta^4 + \frac{3\varkappa+2}{5} \eta^5 \right] - \frac{m^2 \varepsilon D_1 (5\varkappa+2)(3\varkappa+2)}{2(3\varkappa+2)} \times$$

$$\left. \times \left[\frac{1}{2} \eta^2 - \frac{3\varkappa+2}{3} \eta^3 + \frac{2\varkappa+1}{4} \eta^4 \right] \right. . \tag{35}$$
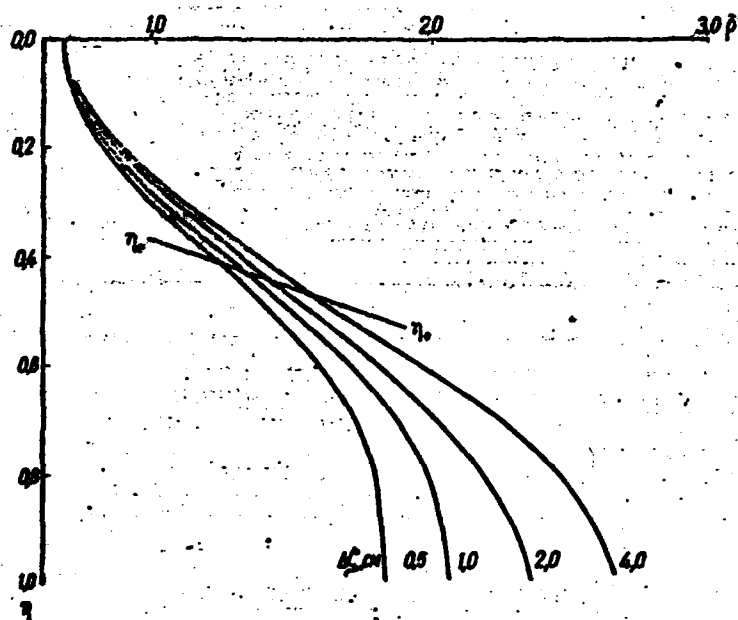

FIG. 4. The Effect of Water Level Decrease on Vertical Distribution of Density Deviation When $\omega=0$ and D_2=const. (line $\eta_*-\eta_*$ shows the depth of interface between water masses).

Figure 4 illustrates the vertical distribution of density deviation. With an increase in the slope of the surface directed against the density gradient $(\varkappa \leqslant 0)$, the interface between water masses lowers. The depth of this surface for various $\varkappa$ is shown in Fig. 4 by line $\eta_*-\eta_*$. In addition, the degree of water stratification increases

with an increase in the slope of water level. Noteworthy is the
fact that wind affects stratification in a similar way, but it   /30
does not lead to a lowering of interface.

The amplitude and character of the influence exercised by various
factors (wind, longitudinal density gradient, and water level
slope) on the vertical current field and density correspond to our
concepts on these phenomena, formed by actual observations.

Application of Automatic Model Solutions
to Concrete Conditions in Bosporus and Other Straits

The solution discussed in this paper enables us to investigate the
problem not in a general form but with special designations of
functions $Q'(\xi)$ and $\bar{h}(\xi)$. With the aid of a known law of density
distribution at water surface and the depth values taken from bathy-
metric charts describing the ends of a given strait it is possible
to determine five unknown parameters of the problem, namely: D_2,
ξ_1, ξ_2, m and α. For this purpose, one has to solve the following
system of equations:

$$
\left.
\begin{aligned}
\bar{\rho}'\,(\xi_1,\ 0) &= s^2 m^{-3}\xi_1^{5\alpha+2} D_2 \\
\bar{\rho}'\,(\xi_2,\ 0) &= s^2 m^{-3}\xi_2^{5\alpha+2} D_2 \\
\bar{h}_1 &= m\xi_1^{-\alpha} \\
\bar{h}_2 &= m\xi_2^{-\alpha} \\
\xi_2 - \xi_1 &= 1
\end{aligned}
\right\}. \qquad (36)
$$

The first two equations of (36) are based on (28) defining two
density values at the surface of the ends of a given strait. The
next two are based on (22). The last equation denotes that a
dimensionless length of strait equals 1. Thus, in order to find
the unknown parameters, it is necessary to know the mean depth at
the end of straits and have data on density values at the ends of
the strait for the meteorological situation associated with the
given problem. In practice, the determination of the general slope
of the bottom for most of the straits and the selection of extreme
depth values appear to be most difficult.

Merz (1928) and V. L. Lebedev (Zubov, 1956) determined the areas
of transverse cross sections in various places of Bosporus and
calculated the mean depths. These calculations enabled us to
determine the values of $\bar{h}_1$ and $\bar{h}_2$. Analogous magnitudes for Bab-el-
Mandeb Strait could be determined only approximately by using Sea

Atlas (1953). It was assumed that the north end of the strait
lies over a threshold near Cape Raheita (see Table 1).

The complex bottom relief of the Strait of Gibraltar obstructed
selection of the general bottom slope. Therefore, in lieu of
determining α and m with the aid of bottom profile, these parameters /31
were determined by means of relationships between density deviations
at bottom and surface and the mean depth at one end of the strait.
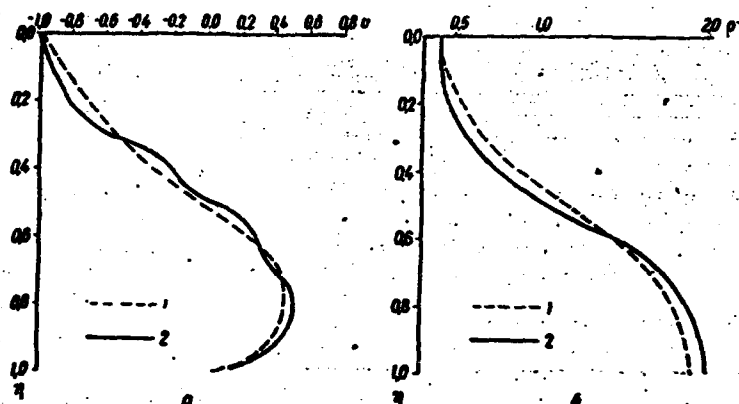These relationships are based on observations and written as follows;

Рис. 5. Сопоставление рассчитанных (1) и наблюденных (2) (Merz, 1928)
скоростей (а) и отклонений плотности (б) для пролива Босфор.

FIG. 5. Comparison of Calculated (1) and Observed (2) (Merz,
1928) Velocities (a) and Deviations of Density (6) for Bosporus.

Fig. 5 demonstrates the relation between observed and calculated
values of current velocities and vertical curves of density deviations
for the Bosporus. The number of such cases for various straits
where calculations correspond to observed data can be increased.

With the aid of the calculation, the wedging of water masses in a
strait, which results from diffusion processes, can be found. For
this purpose, one must find the interface between the surface and
bottom water. It is known that the penetration of saline water of

the bottom current in front of the strait depends upon the intensity
of mixing taking place in the strait. The interface between two
water masses in a strait (h_*) is usually associated with an isopycnic
plane in longitudinal cross section (e.g., $\rho = 1.020$ g/cm^3 in the
Bosporus). By using (35), the position of the surface of cross
section for each strait can be calculated. Then the depth drop of
this surface can be calculated with the aid of the length of the
strait Δh_*. In Table 2 are presented the calculated and observed   /32
magnitudes of Δh_* by making use of the isopycnic plane.

TABLE 2

Strait	Depth Drop	
	Calculated	Observed
Bosporus, $\rho = 1.020$	33	36.5
Bab-el-Mandeb $\rho = 1.0205$	127	110° 160°*
Gibraltar $\rho = 1.028$	379	420°

NOTE:
  *Data by Schott, 1942
 **Data by G. Neumann and McGill

Observations conducted in straits
have made it possible to determine
that the boundary between water
masses has a greater slope than
the boundary between currents.
This incongruence probably depends
on differences between diffusion
flow and the quantity of motion.
Theoretical calculations confirm
it. The depth of zero speed,
h_c, is determined by vertical
profiles of current velocities,
which have been calculated for
several points in a given strait.
According to calculations under
the specified conditions, Δh_c (depth h_c decrease in the strait) is
12m in the Bosporus, 52^m in Bab-el-Mandeb and 22m in Gibraltar. The
observed difference between Δh_c in the Bosporus is 18m (Mars, 1928).
For other straits we do not have such data. According to assumption
by Defant (1961), Δh_c for Bab-el-Mandeb is 40m, for Gibraltar 15m.

The noted difference between calculated and observed magnitudes of
Δh_* and Δh_c probably result from our assumption that coefficients
of viscosity and diffusion are constant. In the last analysis,
the position of boundaries h_* and h_c depends on diffusion flow and
quantity of motion, respectively. Each of these processes depends,
in turn, upon the gradient of the corresponding property in vertical
direction and the magnitude of coefficients. Our assumption that
coefficients μ and k are constant introduces errors into the values
of density and current gradients, which are superimposed over the
errors inherent in the coefficients themselves. Despite this dis-
advantage, the h_* and h_c values (calculated and observed) differ
little. Therefore, the suggested theory appears to be useful.

The resultant solutions enable us to clarify a very important
practical question: under what conditions is the lower current

absent in straits? Ulliot and Ilgaz (1946) doubted that the
lower Bosporus Current is constant. Before them R. de Buen
(1929) proved that the influx of mediterranean water into the   /33
Atlantic Ocean ia an episodic phenomenon.

By using the Bosporus as an example, let us find out under what
conditions the bottom current ceases to exist in the strait.
Under average conditions, the water level decrease in the 30km
stretch of the strait is 6cm. This situation corresponds to our
problem that $\chi = 0.05$. Calculations demonstrate that with $\chi = 0.3$,
the bottom current has a very small velocity. Consequently, in
order to eliminate the two-level system of currents in the Bosporus,
the sea level decrease between the Black and Marmara Seas must
exceed 36cm. Direct observations have demonstrated (Smith, 1946)
that such a slope seldom occurs, and it does not represent a steady
state. This confirms the concept of many oceanographers
(Vodyanitskiy, 1948; Bogdanova, 1959; Pektas, 1959) namely, that
the influx of Marmara Sea water into the Black Sea is continuous.

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13. ABSTRACT

Two-layer water stratification in straits, such as the Bosporus, Bab-el-Mandeb, and Gibraltar, is discussed. Conclusions, based on observed data, are compared with the results of mathematical calculations by considering the position of the interface between the surface and bottom currents, their velocities, slopes and other phenomena. The two opposite currents appear to be active constantly, although the velocity of the bottom current sometimes is extremely small or almost nonexistent. When this occurs, the water-level decrease in the Bosporus, between the Marmara and Black Seas, should exceed 36cm. According to observations and calculations, such cases are rare exceptions and are limited to brief periods.

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